

Closing Tue: TN 3, TN 4 Closing Thu: TN 5

Entry Tasks (Sigma Notation Practice)

1. Differentiate and integrate:

$$f(x) = \sum_{k=2}^5 \frac{(-1)^k}{k^3} x^k = \frac{1}{8}x^2 - \frac{1}{27}x^3 + \frac{1}{64}x^4 - \frac{1}{125}x^5$$

2. Combine

$$5 \sum_{k=2}^4 k^2 x^k - 6 \sum_{k=2}^4 \frac{1}{k!} x^k$$

TN 5: Using Taylor Series

Here are the 6 series you can quote:

$$e^x = \sum_{k=0}^{\infty} \frac{1}{k!} x^k, \quad \text{for all } x$$

$$\sin(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1}, \quad \text{for all } x$$

$$\cos(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k}, \quad \text{for all } x$$

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k, \quad \text{for } -1 < x < 1$$

$$-\ln(1-x) = \sum_{k=0}^{\infty} \frac{1}{k+1} x^{k+1}, \quad -1 < x < 1$$

$$\arctan(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} x^{2k+1}, \quad -1 < x < 1$$

Tools for using Taylor Series

1. Substitute (replace x)

2. Integrate

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C$$

3. Differentiate

$$\frac{d}{dx} (x^n) = nx^{n-1}$$

4. Combine

$$\sum_{k=0}^{\infty} kx^k - 3 \sum_{k=0}^{\infty} \frac{1}{k!} x^k = \sum_{k=0}^{\infty} \left(k - \frac{3}{k!} \right) x^k$$

Substitution Questions: Find the Taylor series based at 0, find the first three nonzero terms and give the interval of convergence.

$$(a) f(x) = 3e^{2x}$$

$$(b) g(x) = \frac{5}{1-4x}$$

$$(c) h(x) = \frac{3}{2x+1}$$

Combining: Find the Taylor series based at 0, find the first three nonzero terms and give the interval of convergence

$$(a) y = 7 + 3x^5 e^{2x}$$

$$(b) y = \frac{5}{1-4x} - \frac{3}{2x+1}$$

(c) $y = \cos^2(x)$ (Hint: Half-angle)

Integrating Applications

(a) Give the first three nonzero terms of the Taylor Series for

$$\int_0^x 7 + 3t^5 e^{2t} dt$$

(b) Find a Taylor series for:

$$A(x) = \int_0^x \frac{\sin(t)}{t} dt$$